

Horn clauses

- A **literal** is an atomic formula or its negation
- A **clause** is a disjunction of literals
- A **Horn clause** is a clause with exactly one positive literal
- A **Horn formula** is a conjunctive normal form formula whose clauses are all Horn

Example

- Prolog:

```
c:- a, b.  
a.  
b.
```

- Horn formula:

$$[c \vee \neg a \vee \neg b] \wedge a \wedge b$$
$$[c, \neg a, \neg b] \quad [a] \quad [b]$$

Example

- Horn formula:

$[c, \neg a, \neg b]$ $[a]$ $[b]$

- Let us attempt to prove c by contradiction
- Hence, the goal clause is $\neg c$

$[c, \neg a, \neg b]$ $[a]$ $[b]$ $[\neg c]$

- By resolution we obtain the empty clause, and hence proof c

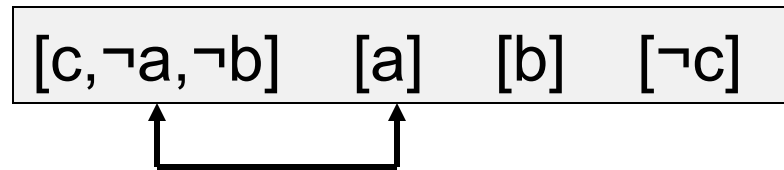
Resolution

- Resolution is a single inference rule
- It takes two clauses, and produces one new clause
- The new clause is implied by the two old clauses
 - The two old clauses need to have complementary literals
 - The new clause contains all the literals of both old clauses except the complementary ones
- Terminates when the empty clause is produced, i.e., a proof has been found

Resolution example

[c, $\neg a$, $\neg b$] [a] [b] [$\neg c$]

Resolution example



Resolution example

[c, $\neg a$, $\neg b$] [a] [b] [$\neg c$]



[c, $\neg b$] [b] [$\neg c$]

Resolution example

$[c, \neg a, \neg b]$ $[a]$ $[b]$ $[\neg c]$



$[c, \neg b]$ $[b]$ $[\neg c]$



Resolution example

$[c, \neg a, \neg b]$ $[a]$ $[b]$ $[\neg c]$



$[c, \neg b]$ $[b]$ $[\neg c]$



$[c]$ $[\neg c]$

Resolution example

$[c, \neg a, \neg b]$ $[a]$ $[b]$ $[\neg c]$



$[c, \neg b]$ $[b]$ $[\neg c]$



$[c]$ $[\neg c]$



Resolution example

$[c, \neg a, \neg b]$ $[a]$ $[b]$ $[\neg c]$



$[c, \neg b]$ $[b]$ $[\neg c]$



$[c]$ $[\neg c]$



$[\]$

Why Horn clauses?

- Resolution of two Horn clauses always results in a Horn clause
- Resolution of a goal clause and a definite clause is always a goal clause
- Horn clauses have better computational properties than normal clauses
- Prolog is based on computing with Horn clauses

Alfred Horn

- The name *Horn clause* comes from **Alfred Horn**, who discovered the significance of such clauses